## A térbeli csavarvonal differenciálgeometriája

$\boldsymbol{r}(t) = R\cos t\,\boldsymbol{i} + R\sin t\,\boldsymbol{j} + at\,\boldsymbol{k}$ *(a csavarvonal tengelye a z tengely.)*

$$\boldsymbol{r} = \begin{bmatrix} R\cos t \\ R\sin t \\ at \end{bmatrix} \qquad \dot{\boldsymbol{r}} = \begin{bmatrix} -R\sin t \\ R\cos t \\ a \end{bmatrix} \qquad \ddot{\boldsymbol{r}} = \begin{bmatrix} -R\cos t \\ -R\sin t \\ 0 \end{bmatrix} \qquad \dddot{\boldsymbol{r}} = \begin{bmatrix} R\sin t \\ -R\cos t \\ 0 \end{bmatrix}$$

$$\dot{\boldsymbol{r}}\times\ddot{\boldsymbol{r}} = \begin{bmatrix} aR\sin t \\ -\quad aR\cos t \\ R^2 \end{bmatrix} \qquad (\dot{\boldsymbol{r}}\times\ddot{\boldsymbol{r}})\times\dot{\boldsymbol{r}} = -R(R^2+a^2)\begin{bmatrix} \cos t \\ \sin t \\ 0 \end{bmatrix} \qquad \dot{\boldsymbol{r}}\ddot{\boldsymbol{r}}\dddot{\boldsymbol{r}} = aR$$

$$|\dot{\boldsymbol{r}}| = \sqrt{R^2+a^2} \qquad |\dot{\boldsymbol{r}}\times\ddot{\boldsymbol{r}}| = R\sqrt{R^2+a^2}$$

érintő egységvektor $\quad \boldsymbol{t} = |\boldsymbol{r}'| = \dfrac{\dot{\boldsymbol{r}}}{|\dot{\boldsymbol{r}}|} = \dfrac{1}{\sqrt{R^2+a^2}}\begin{bmatrix} -R\sin t \\ R\cos t \\ a \end{bmatrix}$

(fő)normális egységvektor $\quad \boldsymbol{n} = \dfrac{(\dot{\boldsymbol{r}}\times\ddot{\boldsymbol{r}})\times\dot{\boldsymbol{r}}}{|\dot{\boldsymbol{r}}\times\ddot{\boldsymbol{r}}|\,|\dot{\boldsymbol{r}}|} = \begin{bmatrix} -\cos t \\ -\sin t \\ 0 \end{bmatrix}$

binormális egységvektor $\quad \boldsymbol{b} = \dfrac{\dot{\boldsymbol{r}}\times\ddot{\boldsymbol{r}}}{|\dot{\boldsymbol{r}}\times\ddot{\boldsymbol{r}}|\,|\dot{\boldsymbol{r}}|} = \dfrac{1}{\sqrt{R^2+a^2}}\begin{bmatrix} a\sin t \\ -\quad a\cos t \\ R \end{bmatrix}$

Görbület:

$$\varkappa = \frac{R}{R^2 + a^2}$$

Csavarodás, torzió:

$$\tau = \frac{a}{R^2 + a^2}$$

Darboux-vektor:

$$\boldsymbol{d} = \tau \boldsymbol{t} + \varkappa \boldsymbol{b} = \frac{1}{\sqrt{R^2 + a^2}} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Ha adott a görbület ($\varkappa$) és a csavarodás ($\tau$): $R = \dfrac{\varkappa}{\varkappa^2 + \tau^2}$ $\quad a = \dfrac{\tau}{\varkappa^2 + \tau^2}$

$$\boldsymbol{t} = \frac{1}{\sqrt{\varkappa^2 + \tau^2}} \begin{bmatrix} -\varkappa \sin t \\ \varkappa \cos t \\ \tau \end{bmatrix} \qquad \boldsymbol{n} = \begin{bmatrix} -\cos t \\ -\sin t \\ 0 \end{bmatrix} \qquad \boldsymbol{b} = \frac{1}{\sqrt{\varkappa^2 + \tau^2}} \begin{bmatrix} -\tau \sin t \\ -\tau \cos t \\ \varkappa \end{bmatrix}$$

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